TSRT14: Sensor Fusion
Lecture 10

— Guest lectures
— Course summary
— The exam

Gustaf Hendeby

gustaf.hendeby@liu.se

LINKÖPING UNIVERSITY

Le 10: guest lectures

Guest lectures:

- Torbjörn Crona & Fredrik Neregård
  — Saab Dynamics
- Roberto Castro Sundin
  — Ericsson

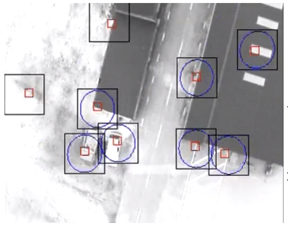
Slides:

- Course summary
- Exam

Lecture 9: summary

Simultaneous Localization And Mapping (SLAM)

- Joint estimation of trajectory $x_{1:k}$ and map parameters θ in sensor model $y_k = h(x_k; \theta) + e_k$.
- Algorithms:
  - EKF-SLAM: EKF (information form) on augmented state vector $z_k = (x_k^T, \theta^T)^T$.
  - FastSLAM: MPF on augmented state vector $z_k = (x_k^T, \theta^T)^T$.
  - GraphSLAM: Formulate the problem as a graph, and use dedicated solvers.



http://youtu.be/VQlaCHl3Yc4

http://youtu.be/hA_NZeuoy9Y

Course Summary

LINKÖPING UNIVERSITY

Parameter Estimation: least squares (LS)

Linear Model

$$\mathbf{y} = \mathbf{H}x + \mathbf{e}, \quad \text{cov}(\mathbf{e}) = \mathbf{R}.$$

- WLS minimizes the loss function

$$V^{\text{WLS}}(x) = (\mathbf{y} - \mathbf{H}x)^T \mathbf{R}^{-1} (\mathbf{y} - \mathbf{H}x).$$

- WLS solution

$$\hat{x} = \big(\mathbf{H}^T \mathbf{R}^{-1} \mathbf{H}\big)^{-1} \mathbf{H}^T \mathbf{R}^{-1} \mathbf{y}, \quad P = \big(\mathbf{H}^T \mathbf{R}^{-1} \mathbf{H}\big)^{-1}.$$

Parameter Estimation: least squares (LS)

Nonlinear Model

$$\mathbf{y} = \mathbf{h}(x) + \mathbf{e}, \quad \text{cov}(\mathbf{e}) = \mathbf{R}.$$

- NWLS minimizes the loss function

$$V^{\text{NWLS}}(x) = \big(\mathbf{y} - \mathbf{h}(x)\big)^T \mathbf{R}^{-1} \big(\mathbf{y} - \mathbf{h}(x)\big)$$

- Solve using optimization methods

$$\hat{x}^{\text{NWLS}} = \arg\min_x V^{\text{NWLS}}(x)$$

Parameter Estimation: nonlinear transform

Approximate $z = g(u)$, $u \sim \mathcal{N}(\hat{u}, P_u)$ with $z \sim \mathcal{N}(\hat{z}, P_z)$.
Variations: TT1, TT2, UT, or MCT.

- The *direct approach*, where $x = \mathbf{h}^{-1}(\mathbf{y} - \mathbf{e})$ is approximated.
- The *indirect approach*, where the distribution of $\mathbf{y} = \mathbf{h}(x)$ is approximated using a prior of $x \sim \mathcal{N}(\hat{x}, P^{xx})$: The trick is to consider the mapping

$$u = \begin{pmatrix} x \\ e \end{pmatrix} \sim \mathcal{N}\left(\begin{pmatrix} \bar{x} \\ 0 \end{pmatrix}, \begin{pmatrix} P^{xx} & 0 \\ 0 & R \end{pmatrix}\right)$$

$$z = \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} x \\ h(x,e) \end{pmatrix} \sim \mathcal{N}\left(\begin{pmatrix} \bar{x} \\ \bar{y} \end{pmatrix}, \begin{pmatrix} P^{xx} & P^{xy} \\ P^{yx} & P^{yy} \end{pmatrix}\right)$$

and then apply

$$\hat{x} = \bar{x} + P^{xy}\big(P^{yy}\big)^{-1}\big(y - \bar{y}\big),$$
$$\text{cov}(\hat{x}) = P^{xx} - P^{xy}\big(P^{yy}\big)^{-1} P^{yx}.$$

Fusing Estimates

- The fusion formula for two **independent estimates** is

$$\mathsf{E}(\hat{x}_1) = \mathsf{E}(\hat{x}_2) = x, \quad \text{cov}(\hat{x}_1) = P_1, \quad \text{cov}(\hat{x}_2) = P_2 \Rightarrow$$
$$\hat{x} = P\big(P_1^{-1}\hat{x}_1 + P_2^{-1}\hat{x}_2\big), \quad P = \big(P_1^{-1} + P_2^{-1}\big)^{-1}.$$

- If the estimates are **not independent**, P is larger than indicated. **Use Safe fusion!**

Sensor Models

- The basic network measurements:

  TOA $r_k = \|x - p_k\| + e_k$
  TDOA $r_k = \|x - p_k\| + r_0 + e_k$
  DOA $\varphi_k = \text{arctan2}(x_2 - p_{k,2}, x_1 - p_{k,1}) + e_k$
  RSS $y_k = P_0 - \beta \log(\|x - p_k\|)$
- Common denominator, all measurements can be described by

$$y_k = h(x; \theta) + e_k.$$

Detection Theory

- Detection problems as hypothesis tests:

$$H_0: \ \mathbf{y} = \mathbf{e},$$
$$H_1: \ \mathbf{y} = \bar{x} + \mathbf{e} = \mathbf{h}(x) + \mathbf{e}.$$

- Neyman-Pearson's lemma:

$$T(y) = p_{\mathbf{e}}\big(y - \mathbf{h}(x^0)\big) / p_{\mathbf{e}}(y)$$

  maximizes P_{D} for given P_{FA} (best ROC curve).
- In general case

$$T(y) = 2 \log p_{\mathbf{e}}\big(y - \mathbf{h}(x^{\text{ML}})\big) - 2 \log p_{\mathbf{e}}(y) \sim \chi^2_{n_x}\big(x^{0,T} \mathcal{I}(x^0) x^0\big).$$

Bayesian Filtering

- Bayes optimal filter:

$$p(x_k | y_{1:k}) \propto p_{e_k}\big(y_k - h(x_k)\big) p(x_k | y_{1:k-1})$$
$$p(x_{k+1} | y_{1:k}) = \int p_{v_k}\big(x_{k+1} - f(x_k)\big) p(x_k | y_{1:k}) \, dx_k.$$

- Intuitive two-stroke work flow of filtering:
  - **MU:** estimation from $y_k = h(x_k) + e_k$ and fusion with $\hat{x}_{k|k-1}$.
  - **TU:** nonlinear transformation $z = f(x_k)$ and diffusion from $x_{k-1} = z_k + v_k$.

Motion Models

- Standard models in global coordinates:
  - Translation $p_t^{(m)} = w_t^p$
  - 2D orientation for heading $h_t^{(m)} = w_t^h$
  - Coordinated turn model

$$\dot{X} = v^X \qquad \dot{Y} = v^Y$$
$$\dot{v}^X = -\omega v^Y \qquad \dot{v}^Y = \omega v^X$$
$$\dot{\omega} = 0$$

- Standard models in local coordinates (x, y, ψ)
  - Odometry and dead reckoning for (x, y, ψ)

$$X_t = X_0 + \int_0^t v_\tau^x \cos(\psi_\tau) \, d\tau \qquad Y_t = Y_0 + \int_0^t v_\tau^x \sin(\psi_\tau) \, d\tau$$
$$\psi_t = \psi_0 + \int_0^t \dot{\psi}_\tau \, d\tau$$

Filtering: Kalman filter (approximations)

Key tool for a unified derivation of KF, EKF, UKF.

$$\begin{pmatrix} X \\ Y \end{pmatrix} \sim \mathcal{N}\left(\begin{pmatrix} \mu_x \\ \mu_y \end{pmatrix}, \begin{pmatrix} P_{xx} & P_{xy} \\ P_{xy} & P_{yy} \end{pmatrix}\right)$$
$$\Rightarrow (X|Y=y) \sim \mathcal{N}(\mu_x + P_{xy}P_{yy}^{-1}(y-\mu_y), P_{xx} - P_{xy}P_{yy}^{-1}P_{yx}).$$

The Kalman gain is in this notation given by $K_k = P_{xy}P_{yy}^{-1}$.

- In KF, P_{xy} and P_{yy} follow from a linear Gaussian model.
- In EKF, P_{xy} and P_{yy} can be computed from a linearized model (requires analytic gradients).
- In EKF and UKF, P_{xy} and P_{yy} computed by NLT for transformation of $(x^T, v^T)^T$ and $(x^T, e^T)^T$, respectively. No gradients required, just function evaluations.

Filtering: SIS PF Algorithm

Choose the number of particles N, a proposal density $q(x_k^{(i)}|x_{0:k-1}^{(i)}, y_{1:k})$, and a threshold N_{th} (for instance $N_{\text{th}} = \frac{2}{3}N$).

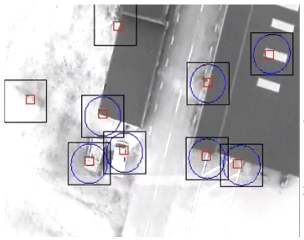
- *Initialization:* Generate $x_0^{(i)} \sim p_{x_0}, i = 1, \dots, N$.

Iterate for $k = 1, 2, \dots$:

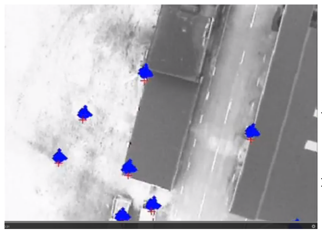
1. *Measurement update:* For $i = 1, 2, \dots, N$: $w_{k|k}^{(i)} = w_{k|k}^{(i)} p(y_k|x_k^{(i)})$, and normalize $w_{k|k}^{(i)}$.
2. *Estimation:* MMSE $\hat{x}_{k|k} \approx \sum_{i=1}^N w_k^{(i)} x_{k|k}^{(i)}$.
3. *Resampling:* Resample with replacement when $N_{\text{eff}} = \frac{1}{\sum_i (w_{k|k}^{(i)})^2} < N_{\text{th}}$.
4. *Prediction:* Generate samples $x_{k+1}^{(i)} \sim q(x_k|x_{k-1}^{(i)}, y_k)$, update the weights $w_{k+1|k}^{(i)} \propto w_{k|k}^{(i)} \frac{p(x_k^{(i)}|x_{k-1}^{(i)})}{q(x_k^{(i)}|x_{k-1}^{(i)}, y_k)}$, and normalize $w_{k+1|k}^{(i)}$.

Simultaneous Localization And Mapping (SLAM)

- Joint estimation of trajectory $x_{1:k}$ and map parameters θ in sensor model $y_k = h(x_k; \theta) + e_k$.
- Algorithms:
  - NLS SLAM: iterate between filtering and mapping.
  - EKF-SLAM: EKF (information form) on augmented state vector $z_k = (x_k^T, \theta^T)^T$.
  - FastSLAM: MPF on augmented state vector $z_k = (x_k^T, \theta^T)^T$.



http://youtu.be/VQlaCH13Yc4



http://youtu.be/hA_NZeuoy9Y

The Exam

The Exam

- The exam is a mixture of pen-and-paper exercises (theory) and computer aided exercises (practical craftsmanship).
- Open book to mimic real life; hence, expect to possibly have to look up details.
- Previous examples available from the homepage:
  `http://www.control.isy.liu.se/student/tsrt14/exam.html`

The Exam: allowed "tools"

Material to bring:

- The course book: *Statistical Sensor Fusion* by F. Gustafsson.

Normal study notes in the books are okay!

Made available digitally:

- A current errata for the textbook
- Lecture slides for 2023
- Signal and Systems toolbox manual

The Exam: practicalities

- Performed in the ISY computer labs.
- Clean account: Matlab, SigSys, and required data available.
- Watch out when printing to remain anonymous.

The Exam: advice

General Advice

- Read through all exercises and **prioritize**, before getting started.
- Make sure to **motivate** every step of your solution carefully!
- **Comment** nontrivial steps in any code you write; including model choices and tuning
- **Separate papers** for all exercises, keep the papers for each exercise together.

GOOD LUCK!